Linear Finite Element Algorithm on Contact Stress of Bogie Wheels and Crawler Belts

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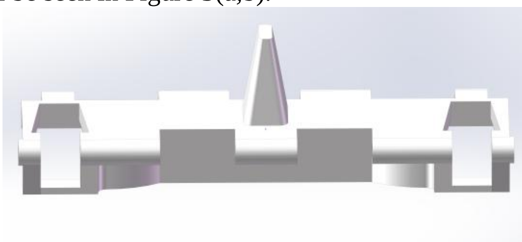
Abstract: In tracklayers, the contact of bogie wheels and crawler belts involves multiple fields including geometry, materials and contact nonlinearity. In this paper, the elastic contact equation of bogie wheels and crawler belts was first established, and then the linear finite element algorithm was used to construct the influence coefficient stiffness matrix. Next, MATLAB programming was adopted to calculate the numerical value, positions and distributed computing results of the contact stress between bogie wheels and crawler belts. The results computed though MATLAB were approximate to those of engineering surveying. It proved that the linear finite element algorithm was of engineering reference value when the requirements of the engineering precision are met.

Keywords: bogie wheels, crawler belts, contact stress, linear finite element

1. Introduction

Tracklayers can be applied in relatively complex road conditions [1]. Bogie wheels and crawler belts are important components responsible for the motion of tracklayers. Crawler belt is made of high manganese steel [2] with an elasticity modulus of 200,000 Mpa and a Poisson ratio of 0.29. The smooth surface is in contact with bogie wheels, while the grouser surface is in contact with the ground because grousers can effectively increase the adhesive forces of the road surface, as can be seen in Figure 1(a,b). Bogie wheels are made up of the left wheel and the right wheel, and plastic materials are distributed on the surface, which helps to reduce the plastic deformation caused by the impact of bogie wheels and crawler belts. Plastic materials are 90,000 Mpa in elasticity modulus, 0.32 in Poisson ratio, and bogie wheels can effectively spread the total weight of vehicles, as can be seen in Figure 2(a,b). By calculating the contact stress distribution of bogie wheels and crawler belts, the research is of great significance in studying the terramechanics of tracklayers. The weight of the tracklayer body works on bogie wheels, which can evenly distribute the gravitational load of the tracklayer body [3]. Based on the structure of bogie wheels, the tracklayer body load can be equivalent to the concentrated force load, and the former vertically acts on the midpoint of the axis between two sub-wheels of the bogie wheels. The contact domain is approximate to the oval or the rectangle, as can be seen in Figure 3(a,b).



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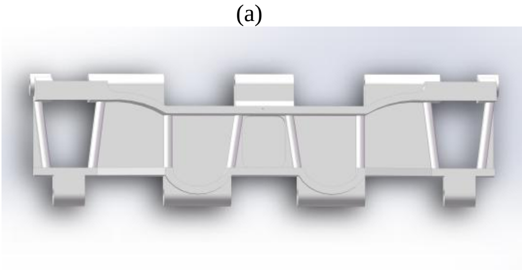

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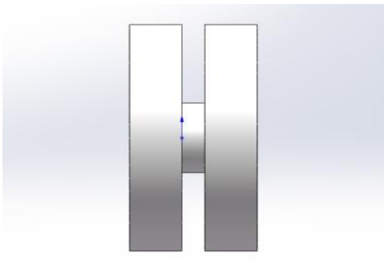
Figure 1. 3-D model diagram of crawler belts



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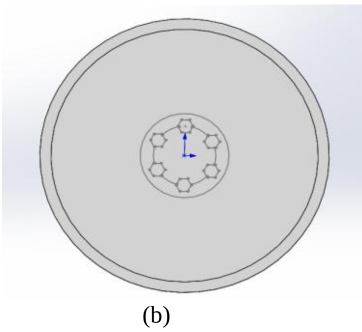

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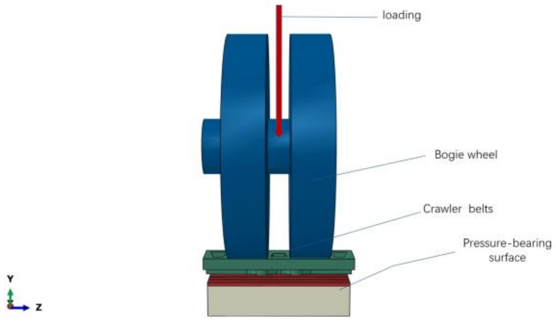
Figure 2. 3-D model diagram of bogie wheels



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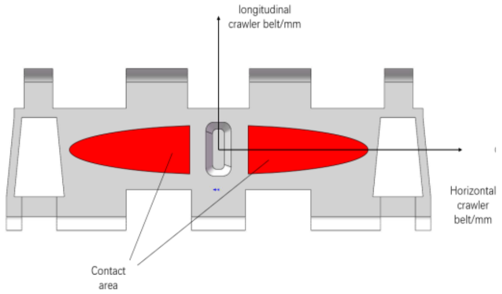

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Figure 3. simplified sketch map of the model

2. Theory of Contact Stress Computation

The contact between bogie wheels and crawler belts conforms to the hertz contact theory. Namely, three assumed conditions of the hertz contact theory should be satisfied.

(1) Only elasticity deformation is made in the contact objects, and Hooke's Law should be followed.

(2) The load is vertical to the contact surface.

(3) The size of the contact surface is smaller than that of the contact object's surface.

Hertz contact theory [4] was put forward by Hertz over 10 years ago, which provides a theoretic solution for the elastic contact question of point contact and line contact. With the popularization of computers, considerable contact questions can be solved by using the finite element method. The Based on the elasticity mechanics and the linear finite element method, the influence coefficient matrix was constructed to compute the contact stress that can satisfy the regulated iteration accuracy in this paper [5].

$$\iint_S P(x,y)dxdy = F \ . \tag{1}$$

$$\frac{1}{\pi E^{'}}\iint_S \frac{P(x,y)dxdy}{\sqrt{(x-x^{'})^2+(y-y^{'})^2}} = \delta - z(x,y) \ . \tag{2}$$

Formula (1) represents the equilibrium equation, where is the distribution of contact stress. Formula (2) represents the deformation compatibility equation. The integrand function on the left is Boussinnesq solution in elasticity mechanics. It means the displacement generated on (x,y) when the concentrated force works on half space (x,y) , δ is elastic approach, meaning the deformation of normal contact [6]. $z(x,y)$ is the initial distance between two contact surfaces and it can be expressed through the quadratic function.

$$\delta = 4.83\times 10^{-5}\frac{F^{0.9}}{l^{0.74}D^{0.1}} \ . \tag{3}$$

In the formula, l is effective contact length, F is running pulley load, and D is running pulley diameter.

$$z(x_k,y_k) = Ax_k^2 + By_k^2 \ . \tag{4}$$

The contact domain is pre-estimated based on the Hertz contact theory, and then the contact half-width was computed using the formulas (5), which helps to estimate the contact domain. Meanwhile, the semi-contact domain was divided into $M\times N$ small rectangular elements, the side length of which is am and bn respectively. Suppose the contact stress of each element is even, it can be dispersed into the following formula.

$$\begin{cases} 2\sum_{j=1}^{M\times N} a_m b_n P_j = F \\ \frac{1}{\pi E^{'}}\sum_{j=1}^{M\times N} K_{kj}P_j = \delta - z(x_k,y_k) \end{cases} \ . \tag{5}$$

Where K_{kj} is influence coefficient matrix [7,8], meaning the central deformation of the element numbered k, which is caused by P_j . The expression is as follows:

$$K_{kj} = \iint_S \frac{dxdy}{\sqrt{(x-x_k)^2+(y-y_k)^2}} \ . \tag{6}$$

And the following conditions should be satisfied.

$$\begin{cases} x_j - a_m/2 \le x \le x_j + a_m/2 \\ y_j - b_n/2 \le y \le y_j + b_n/2 \\ 1 \le k \le M\times N \end{cases} \ . \tag{7}$$

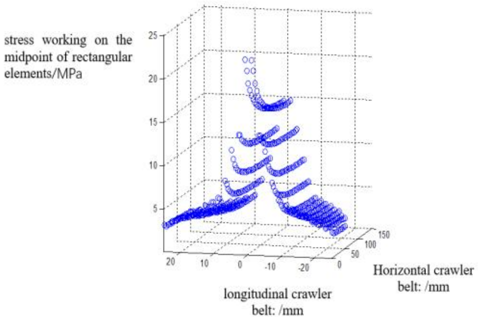
The two-dimensional Gaussian integral method was used to get the solution of the influence coefficient matrix K_{kj} , S integral domain was divided into n^2 small rectangular integral elements, where the side length of small rectangular was the integral weight coefficient W_i, W_j , so the expression (6) above can written as:

$$K_{kj} = \iint_S \frac{dxdy}{\sqrt{(x-x_k)^2+(y-y_k)^2}} = \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \frac{1}{\sqrt{(x_i-x_k)^2+(y_j-y_k)^2}} \ . \tag{8}$$

The semi contact domain was divided into small rectangular elements, so the computation involved the relatively high-order matrix operation. Given the efficiency of MATLAB in computing high-order matrixes, MATLAB programming was used to compute the contact stress of bogie wheels and crawler belts [9].

3. Analysis of MATLAB Computing Results

MATLAB was used to compute the contact stress of the node in the divided $M\times N$ small rectangular elements in the semi contact domain, as seen in Figure 4(a, b).



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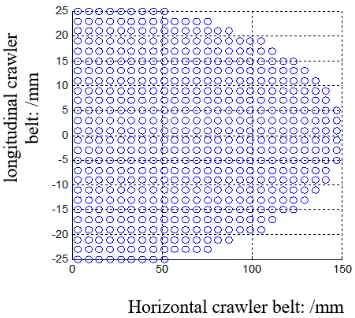


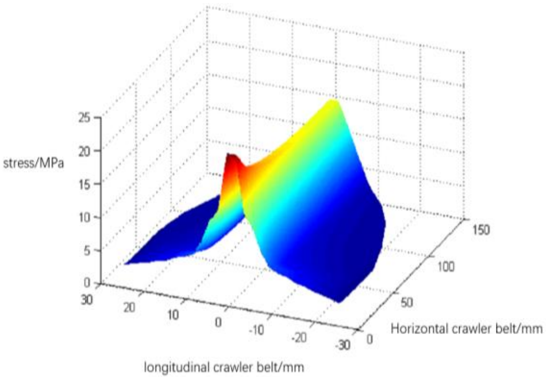
(b)

Figure 4. distribution diagram of contact stress nodes between bogie wheels and crawler belt semi-contact domain

As required, the iteration accuracy of $\varepsilon \le 2\%$ was set and then the contact stress of each node were summated to get the value of F_p. When F_p was compared with the value of the real load FR, it was found that the computed accuracy could satisfy the iteration accuracy.

$$\frac{|F_p - F_{real}|}{F_{real}} = 1.473\% < \varepsilon . \qquad (9)$$

The linear distribution algorithm was used to disperse the stress linearity of each small rectangular element node around the whole rectangular domain. In this way, the stress distribution of the whole contact domain can be seen in Figure 5(a, b).



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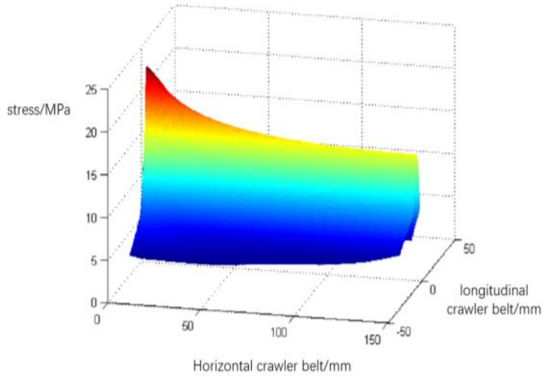

(b)

Figure 5. distribution diagram of contact stress between bogie wheels and semi contact domain of crawler belts

It can be known in Figure 4 and 5 that: the contact stress of bogie wheels and crawler belts was distributed in the oval domain and centered in the longitudinal centre of crawler belts (-10mm, +10mm), where the contact stress was marked by greater numerical value and significant changes. However, the contact stress was smaller in numerical value and changed slowly. At the contact center, the maximum contact stress reached 24.56Mpa, and the contact stress gradually decreased towards the margin along the horizontal crawler belts.

4. Conclusion

This paper computed the numerical value, positions and distribution of contact stress between bogie wheels and crawler belts by constructing the elastic contact equation of bogie wheels and crawler belts as well as the influence coefficient stiffness matrix through MATLAB. It was tested that the computing results of MATLAB were approximate to those of engineering surveying. It proved that the linear finite element algorithm was of engineering reference value when the engineering accuracy conditions are satisfied.

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